

Positive Factorization and a Zero-Energy Resonance of the Schwinger-Coupled Wormhole Mouth

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Abstract

We analyze the three-component extremal-mouth operator obtained by Sadhukhan for the Schwinger-coupled axial-metric/polar-gauge sector of the Maldacena–Milekhin–Popov wormhole. For every multipole $\ell \geq 2$ and every fixed finite screening parameter $M > 0$, we give an exact positive factorization and classify all bounded zero-energy solutions. The operator has no negative spectrum and no zero eigenfunction, but has a unique bounded zero-energy resonance up to normalization. A variational corollary shows that every compactly supported attractive matrix perturbation with nonzero overlap with this resonance creates a negative eigenvalue, however small its strength. We also compute classical finite-frequency scattering with all three outgoing channels and inverse-square endpoint reference functions. A threshold transmission formula follows from a stated matching hypothesis and is supported by the computations; a uniform matrix threshold theorem is not proved. The results concern one specified extended extremal mouth. They do not establish stability of a fully glued wormhole or a physical quantum communication channel.

1 Introduction and relation to prior work

Magnetic traversable wormholes supported by charged-fermion vacuum effects provide a concrete setting in which to ask how excitations propagate through a gravitational background [1]. Transmission through their mouths is a separate issue from geometric traversability [2]. In the axial-metric/polar-gauge sector, the Schwinger response mixes the metric, electric, and charge-density master variables. Sadhukhan derives the three-channel mouth equations, identifies a neutral throat mode, and reports finite-element evidence for the absence of negative mouth eigenvalues [3, Secs. 5, 7, 10]. A negative pointwise eigenvalue of the mouth potential does not by itself settle stability, because its eigenvector varies with radius.

We provide an exact factorization of that full three-channel extremal operator, valid for all fixed finite $M > 0$ and $\ell \geq 2$, together with a closed-form radial resonance and a proof of its uniqueness among bounded zero-energy solutions. The factorization supplies an analytic version of the mouth stability conclusion in this specified model. The resonance also exposes sensitivity to attractive corrections: positivity of the unperturbed operator does not supply a positive margin against every small localized perturbation.

The underlying method is established. Matrix S -deformations are standard tools for coupled black-hole stability [4]; deformations proportional to f/r occur in related Einstein–Maxwell–scalar systems [5]. The contribution here is the explicit Schwinger-coupled completion and its full radial kernel, not a new factorization method or the discovery of the neutral throat eigenvector. A

documented search of the closest primary literature did not locate these exact full-mouth formulas; this is a scoped novelty assessment, not a guarantee of exhaustive priority.

2 The specified extremal-mouth operator

Set the mouth radius and the extremal gravitational length to one. For $r > 1$, define

$$f(r) = \left(1 - \frac{1}{r}\right)^2, \quad x = r + 2\log(r-1) - \frac{1}{r-1}, \quad \frac{dr}{dx} = f. \quad (1)$$

Thus $x \in \mathbb{R}$. The limit $x \rightarrow -\infty$ is the extended extremal inner end. A finite traversable throat requires a different global matching problem. Write

$$L = \ell(\ell+1), \quad \mu = \sqrt{L-2}, \quad M = m_S^2 > 0. \quad (2)$$

Here M is the dimensionless Schwinger screening parameter. It is neither a fermion rest mass nor the mass of the black hole. We take Eqs. (5.20)–(5.22) of Sadhukhan [3] as the model input, at the extremal-mouth order used there. With

$$Z = (\Psi_g, \Psi_D, \Xi)^T, \quad Y = SZ, \quad S = \text{diag}(\mu, 1, \sqrt{LM}), \quad (3)$$

the equation is $HY = k^2Y$, where

$$H = -\partial_x^2 + V, \quad V = f \begin{pmatrix} L/r^2 - 6/r^3 + 4/r^4 & -2\mu/r^3 & 0 \\ -2\mu/r^3 & L/r^2 + 4/r^4 & \sqrt{LM}/r^2 \\ 0 & \sqrt{LM}/r^2 & M/r^2 \end{pmatrix}. \quad (4)$$

For each fixed finite parameter pair, V is real symmetric, smooth, bounded, and tends to zero at both ends as $O(x^{-2})$. Consequently H is self-adjoint on $H^2(\mathbb{R}, \mathbb{C}^3)$, with form domain $H^1(\mathbb{R}, \mathbb{C}^3)$. Multiplication by V is relatively compact with respect to the free resolvent: on a compact interval this follows from local Sobolev compactness, and the exterior multiplication norm tends to zero. It follows that

$$\sigma_{\text{ess}}(H) = [0, \infty). \quad (5)$$

The constant symmetrizer is invertible only in the stated range. We do not extend this construction to $\ell = 0, 1$ or $M = 0$ by assertion.

3 Positive factorization and the zero resonance

Theorem 1 (Exact positive factorization). *For $\ell \geq 2$ and every fixed finite $M > 0$, let*

$$W = \text{diag}(f/r, 0, 0), \quad Q = \partial_x + W, \quad w_1 = (\mu, -2/r, 0)^T, \quad w_2 = (0, \sqrt{L}, \sqrt{M})^T. \quad (6)$$

Then, as an operator identity on smooth compactly supported vectors and as a closed quadratic-form identity,

$$\boxed{H = Q^\dagger Q + \frac{f}{r^2}(w_1 w_1^T + w_2 w_2^T) \geq 0.} \quad (7)$$

In particular, H has no negative spectrum.

Proof. The potential of $Q^\dagger Q$ is $W^2 - W_x$. Its only nonzero entry is

$$\left(\frac{f}{r}\right)^2 - f \frac{d}{dr} \left(\frac{f}{r}\right) = f \left(\frac{2}{r^2} - \frac{6}{r^3} + \frac{4}{r^4}\right). \quad (8)$$

The first rank-one term adds $f\mu^2/r^2$ to the first diagonal entry, $4f/r^4$ to the second, and $-2f\mu/r^3$ to their off-diagonal entries. The second adds fL/r^2 , fM/r^2 , and $f\sqrt{LM}/r^2$ to the remaining block. Since $\mu^2 = L - 2$, their sum is exactly Eq. (4). Integration by parts gives

$$q_H[u] = \|Qu\|_2^2 + \int_{\mathbb{R}} \frac{f}{r^2} (|w_1^T u|^2 + |w_2^T u|^2) dx. \quad (9)$$

All coefficients are bounded; density extends the identity from C_c^∞ to H^1 . Nonnegativity follows. $\square$

Theorem 2 (Unique bounded zero-energy solution). *The bounded solutions of $Hu = 0$ form the one-dimensional space spanned by*

$$u_0(r) = \left(-\frac{2\sqrt{M}}{\mu r}, -\sqrt{M}, \sqrt{L} \right)^T. \quad (10)$$

This solution is not in $L^2(\mathbb{R}, \mathbb{C}^3)$, and H has no zero eigenfunction. We call u_0 a zero-energy resonance, meaning here a bounded non-square-integrable zero-energy solution.

Proof. Direct differentiation gives $Qu_0 = 0$ and $w_i^T u_0 = 0$ for $i = 1, 2$. Equation (7) therefore implies $Hu_0 = 0$. Its last two components are nonzero constants, so it is bounded but not square-integrable.

Conversely, let u be any bounded solution. Since $u'' = Vu = O(x^{-2})$, u' has finite limits at both ends. Boundedness of u forces those limits to be zero, and integration of u'' then yields $u' = O(|x|^{-1})$. Also $W \rightarrow 0$. Applying the factorized identity on $[-R, R]$ gives

$$\int_{-R}^R \left(|Qu|^2 + \frac{f}{r^2} \sum_{i=1}^2 |w_i^T u|^2 \right) dx = \operatorname{Re}[u^\dagger Qu]_{-R}^R \rightarrow 0. \quad (11)$$

The nonnegative integrand must vanish. The equation $Qu = 0$ gives $u_1 = c_1/r$, $u_2 = c_2$, and $u_3 = c_3$; the two remaining constraints are $\mu c_1 = 2c_2$ and $\sqrt{L}c_2 + \sqrt{M}c_3 = 0$. They leave exactly the span of Eq. (10). An L^2 zero eigenfunction belongs to H^2 and is bounded, so the same classification excludes it. $\square$

This is a statement about the frequency-independent master operator. The original master-variable definitions contain frequency factors [3, Eq. (3.5)]. In particular, the neutral combination has $\Psi_D + M\Xi = 0$ and $\Xi \neq 0$. Its existence does not by itself reconstruct a finite static four-dimensional perturbation at exactly zero frequency. The operator resonance and static field reconstruction are distinct questions.

4 Sensitivity to attractive corrections

Corollary 1 (No uniform margin against localized attraction). *Let $B(x)$ be a bounded, compactly supported Hermitian matrix with $B(x) \geq 0$ and*

$$I_B = \int_{\mathbb{R}} u_0^\dagger B u_0 dx > 0. \quad (12)$$

For every $\varepsilon > 0$, the operator $H_\varepsilon = H - \varepsilon B$ has a negative discrete eigenvalue.

Proof. Choose a smooth real cutoff χ_R which equals one on $[-R, R]$, vanishes outside $[-2R, 2R]$, and satisfies $|\chi'_R| \leq C/R$. Since the resonance annihilates all the non-derivative factors in Eq. (9),

$$q_H[\chi_R u_0] = \int_{\mathbb{R}} |\chi'_R|^2 |u_0|^2 dx = O(R^{-1}). \quad (13)$$

Once the support of B lies inside $[-R, R]$,

$$q_{H_\varepsilon}[\chi_R u_0] = O(R^{-1}) - \varepsilon I_B < 0 \quad (14)$$

for sufficiently large R . Compactly supported bounded multiplication is relatively compact with respect to H , so $\sigma_{\text{ess}}(H_\varepsilon) = [0, \infty)$. The variational principle therefore gives a negative discrete eigenvalue. $\square$

The condition $I_B > 0$ matters: an attraction annihilating the resonance is not covered. The corollary does not determine the sign, support, or matrix structure of any actual correction to the wormhole equations. It shows why smallness of unspecified corrections alone cannot extend a zero-threshold stability statement to a different global operator.

5 Endpoint channels and a threshold matching prediction

The endpoint expansions are

$$V(x) = \frac{C_\pm}{x^2} + O\left(\frac{\log|x|}{|x|^3}\right), \quad x \rightarrow \pm\infty, \quad (15)$$

with

$$C_- = \begin{pmatrix} L-2 & -2\mu & 0 \\ -2\mu & L+4 & \sqrt{LM} \\ 0 & \sqrt{LM} & M \end{pmatrix}, \quad C_+ = \begin{pmatrix} L & 0 & 0 \\ 0 & L & \sqrt{LM} \\ 0 & \sqrt{LM} & M \end{pmatrix}. \quad (16)$$

Their null vectors are the endpoint values of the same exact solution:

$$u_- = (-2\sqrt{M}/\mu, -\sqrt{M}, \sqrt{L})^T, \quad a^2 := |u_-|^2 = L + M + \frac{4M}{L-2}, \quad (17)$$

$$u_+ = (0, -\sqrt{M}, \sqrt{L})^T, \quad b^2 := |u_+|^2 = L + M. \quad (18)$$

The normalized neutral channels are $n_- = u_-/a$ and $n_+ = u_+/b$. The remaining eigenvalues of C_+ are L and $L + M$; those of C_- are

$$\lambda_\pm = \frac{2L + 2 + M \pm \sqrt{(M-2)^2 + 16L}}{2} > 0. \quad (19)$$

These reproduce the known endpoint mass matrices [3, Secs. 7–8].

For real $k > 0$, the conserved current is

$$j = \text{Im}(Y^\dagger Y_x) = \text{Im} \left(\mu^2 \Psi_g^* \Psi'_g + \Psi_D^* \Psi'_D + LM \Xi^* \Xi' \right). \quad (20)$$

A unit-amplitude plane wave has current magnitude k . Reflection and transmission fractions below divide by that incoming current. This follows from the symmetric master equation; a physical one-particle normalization would additionally require a reduction of the four-dimensional symplectic form.

For a unit-amplitude neutral wave incident from the left, consider the following *matching hypothesis*: as $k \downarrow 0$ at fixed ℓ, M , the scattering solution has a bounded overlap limit Cu_0 , the outgoing amplitudes in the two nonneutral endpoint channels vanish, and matching errors in its Wronskian with u_0 are $o(k)$. Then its neutral amplitudes satisfy

$$1 + r_0 = Ca, \quad t_0 = Cb, \quad a(1 - r_0) = bt_0. \quad (21)$$

For $Hu_0 = 0$ and $HY = k^2Y$, the bilinear Wronskian $\mathcal{W} = u_0^T Y_x - (u'_0)^T Y$ satisfies $\mathcal{W}' = -k^2 u_0^T Y$; it is not exactly constant at $k > 0$. Under the matching hypothesis its change across the overlap $1 \ll |x| \ll k^{-1}$ is $o(k)$, yielding the last relation. Solving this algebra yields the prediction

$$r_{\text{match}} = \frac{a^2 - b^2}{a^2 + b^2} = \frac{2M}{L(L + M - 2)}, \quad \boxed{T_{\text{match}} = 1 - \left[\frac{2M}{L(L + M - 2)} \right]^2}. \quad (22)$$

The endpoint norms of the exact resonance thus fix the proposed threshold transmission, conditional on the stated limit.

We do *not* prove that uniform matrix threshold limit here. The full potential has an inverse-square tail and does not satisfy the usual finite-first-moment assumption for short-range zero-energy scattering; scalar short-range results such as Egorova et al. [6] cannot be applied directly. Inverse-square threshold theory has separate hypotheses [7]; its existence alone is not a theorem for this resonant coupled system. The residual after subtracting the reference tails in Eq. (15) does have a finite first moment, but one must still control the uniform matrix matching and suppress the nonneutral coefficients. The numerical results below test Eq. (22) without promoting it to a proved limit. Its subsequent $M \rightarrow \infty$ limit is $1 - 4/L^2$, for example 8/9 at $\ell = 2$; no interchange of the M and k limits is asserted.

6 Finite-frequency scattering and reproducibility

We solve the full three-channel equation, keeping all three reflected and all three transmitted amplitudes. The numerical coordinate is $z = \log(r - 1)$, with $dx/dz = e^z + 2 + e^{-z}$. We evolve $(Y, Y_x/k)$ as twelve real variables using adaptive fourth-order collocation. The finite endpoints are $z = \pm \log(E/k)$, where E is an adjustable extent.

For each endpoint eigenvalue $c \geq 0$, the outgoing reference is

$$h_c(x) = \sqrt{\frac{\pi k |x|}{2}} e^{i(\pi\nu/2 + \pi/4)} H_\nu^{(1)}(k|x|), \quad \nu = \sqrt{\frac{1}{4} + c}. \quad (23)$$

It tends to $e^{ik|x|}$ and carries outward current of magnitude k . An orthonormal eigenbasis of $C_\pm$ converts these references to matrix logarithmic-derivative boundary conditions. At the left endpoint the neutral incoming wave supplies an inhomogeneous boundary condition; at the right all channels are outgoing. The calculation returns complex amplitude vectors r, t and

$$R = \sum_{i=0}^2 |r_i|^2, \quad T = \sum_{i=0}^2 |t_i|^2, \quad T_{nn} = |t_0|^2, \quad R + T = 1. \quad (24)$$

The last identity is a conservation check. The amplitudes in all nonneutral outgoing channels are retained even when small.

Endpoint-reference error is distinct from the collocation residual. We therefore vary E independently of solver tolerance and report changes in observables. A small flux defect alone does not

Table 1: Smallest-frequency results at $k = 10^{-5}$ and $E = 40$. T_{match} is a conditional prediction, not an established limit.

ℓ	M	T_{nn}	T_{match}	Difference
2	1	0.995556076	0.995555556	5.20×10^{-7}
2	10	0.943316086	0.943310658	5.43×10^{-6}
2	100	0.897280306	0.897271532	8.77×10^{-6}
3	1	0.999770459	0.999770432	2.78×10^{-8}
3	10	0.993056355	0.993055556	7.99×10^{-7}
3	100	0.977045606	0.977043159	2.45×10^{-6}
5	1	0.999994716	0.999994715	4.77×10^{-10}
5	10	0.999692250	0.999692213	3.74×10^{-8}
5	100	0.997287649	0.997287326	3.22×10^{-7}

Table 2: Absolute changes in T_{nn} under independent refinements.

ℓ	M	k	Tolerance	40 $\rightarrow$ 80	80 $\rightarrow$ 160
2	100	1.00×10^{-5}	1.49×10^{-9}	5.22×10^{-10}	1.83×10^{-10}
2	100	3.00×10^{-1}	1.06×10^{-9}	3.30×10^{-5}	8.58×10^{-6}
3	10	1.00×10^{-2}	4.44×10^{-10}	1.21×10^{-8}	2.32×10^{-9}
5	1	1.00×10^{-4}	1.91×10^{-10}	7.76×10^{-11}	3.33×10^{-10}

bound the error introduced by discarding the residual tails. Additional checks compare left/right neutral transmission amplitudes and an independent initial-value shooting calculation at a moderate parameter point.

The committed grid contains 90 successful solves at $\ell \in \{2, 3, 5\}$, $M \in \{1, 10, 100\}$, and ten frequencies from 10^{-5} to 0.3. The base extent is $E = 40$ and the collocation tolerance is 2×10^{-7} . The maximum all-channel flux defect $|R+T-1|$ on this grid is 1.54×10^{-9} ; the maximum sampled interior normalized-current discrepancy is 1.06×10^{-8} . These diagnostics measure conservation within the finite-domain boundary-value problem, not its full endpoint-truncation error.

Table 2 separates tolerance and endpoint tests. The tolerance column compares 2×10^{-7} with 2×10^{-8} at $E = 40$. Extent columns keep the base tolerance and compare successive $E = 40, 80, 160$ calculations. They are observed changes, not rigorous global error bounds. In particular, finite-frequency endpoint effects can exceed the flux defect by several orders of magnitude.

7 Physical scope and conclusions

The exact factorization, bounded-kernel classification, and attractive perturbation corollary are independent of the finite-frequency calculations. Together they establish a complete spectral statement at and below zero for the specified one-mouth operator. They explain how an indefinite pointwise potential can have a nonnegative Schrodinger operator, and why the resulting zero threshold leaves no universal stability margin against localized attraction.

The model inherits the extremal-mouth and lowest-Landau-level approximations of Sadhukhan [3]. Extending it to a finite two-mouth wormhole requires its throat, matching conditions, boundary conditions, and correction terms. Neither the theorem nor the perturbation corollary settles that global stability problem. The numerical parameter grid also does not probe the very large screening

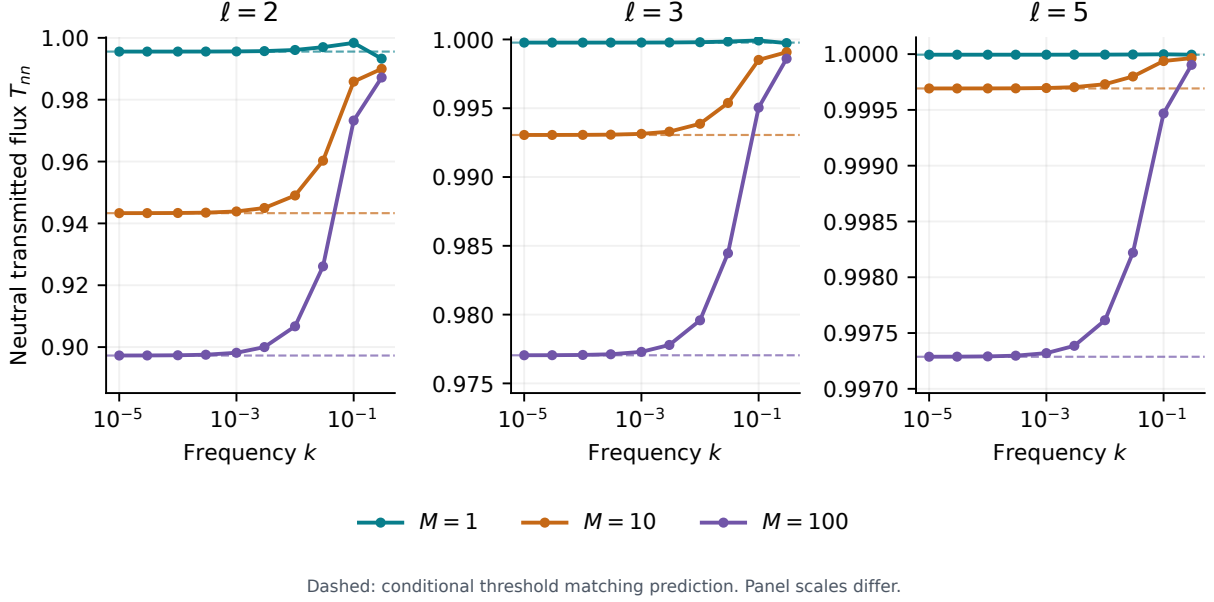


Figure 1: Neutral-to-neutral transmitted master flux for the full three-channel operator. Solid curves join the computed finite-frequency samples; dashed lines are the conditional matching prediction (22). Each panel holds ℓ fixed, and colors label the screening parameter. Numerical approach to a dashed line does not replace a uniform threshold proof. These fractions are classical master-flux observables, not quantum-state fidelities.

values used in particular macroscopic examples.

A physical communication calculation further requires canonically normalized accessible inputs and outputs, a specified receiver, and the environmental channels that distinguish encoded states. The present conserved master current does not supply those ingredients by itself. In particular, finite screening $M > 0$ does not establish a result for nonzero fermion rest mass. The project's broader quantum-coherence hypothesis remains open.

The next mathematical question within this model is whether the inverse-square matrix matching hypothesis can be proved, and with what error bound. The present paper's proved contribution does not depend on an affirmative answer.

Code and data availability

The source, numerical configuration, complex scattering amplitudes, convergence checks, figures, and SHA-256 provenance manifest are in <https://github.com/agent-axiom/teleport-research>. The calculation is reproduced by `make reproduce-neutral`; the theorem notes and literature comparison are in the accompanying `docs` directory. The repository is currently private. A public archival release or equivalent reader access must be arranged before a submission cites it as openly available. The manuscript has no assigned arXiv identifier.

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Authorship and preparation

AI assistance was used for derivations, coding, literature checking, and internal review. These internal checks are not external peer review. The author is responsible for verifying the complete manuscript and for the submitted contents; no AI system is listed as an author.

References

- [1] Juan Maldacena, Alexey Milekhin, and Fedor Popov. Traversable wormholes in four dimensions, 2018. URL <https://arxiv.org/abs/1807.04726v3>. arXiv:1807.04726v3. Version 3, 3 November 2020.
- [2] Ben Freivogel, Alessandro Fumagalli, and Marija Tomašević. How traversable is a traversable wormhole?, 2026. URL <https://arxiv.org/abs/2606.12528v2>. arXiv:2606.12528v2. Version 2, 14 September 2026.
- [3] Abhishake Sadhukhan. Axial-metric/polar-gauge perturbations of the Maldacena-Milekhin-Popov wormhole, 2026. URL <https://arxiv.org/abs/2610.09847v1>. arXiv:2610.09847v1. Submitted 7 October 2026.
- [4] Masashi Kimura and Takahiro Tanaka. Stability analysis of black holes by the S-deformation method for coupled systems, 2018. URL <https://arxiv.org/abs/1809.00795v3>. arXiv:1809.00795v3. Version 3, 25 January 2019.
- [5] Aron Jansen, Andrzej Rostworowski, and Mieszko Rutkowski. Master equations and stability of Einstein-Maxwell-scalar black holes, 2019. URL <https://arxiv.org/abs/1909.04049v2>. arXiv:1909.04049v2. Version 2, 22 April 2021.
- [6] Iryna Egorova, Markus Holzleitner, and Gerald Teschl. Zero Energy Scattering for One-Dimensional Schrodinger Operators and Applications to Dispersive Estimates, 2015. URL <https://arxiv.org/abs/1504.05744v2>. arXiv:1504.05744v2. Version 2, 8 December 2015.
- [7] Ovidiu Costin, Wilhelm Schlag, Wolfgang Staubach, and Saleh Tanveer. Semiclassical analysis of low and zero energy scattering for one dimensional Schrodinger operators with inverse square potentials, 2008. URL <https://arxiv.org/abs/0804.2282v2>. arXiv:0804.2282v2. Version 2, 16 April 2008.